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Solved Paper 2016

GATE Electrical Engineering

Time : 3 hrs **MM :** 100

Read the following instructions carefully

1. There are 65 questions carrying 100 marks of 3 hrs duration in this paper.
2. Questions 1 to 25 carry 1 mark each and questions 26 to 55 carry 2 marks each.
3. Questions 56 to 65 belong to General Aptitude (GA) Type. Questions 56 to 60 carry 1 mark each and questions 61 to 65 carry 2 marks each.
4. Unattempted questions will carry zero marks.
5. For questions 1 to 25 and 56 to 60, 1/3 mark will be deducted for each wrong answer. For questions 26 to 55 and 61 to 65, 2/3 mark will be deducted for each wrong answer.
6. There is no negative marking for numerical answer type questions.
7. Physical calculator, charts, graph sheets and mathematical tables are not allowed in the examination hall.

(1 *Mark Questions*)

1. The maximum value attained by the function $f(x) = x(x-1)(x-2)$ in the interval $[1, 2]$ is _____.

Sol. (0) Given function $f(x) = x(x-1)(x-2)$ in $[1,2]$

$$f(x) = x^3 - 3x^2 + 2x$$

On differentiating w.r.t x, we get

$$f'(x) = 3x^2 - 6x + 2$$

For critical points put $f'(x) = 0$

$$\Rightarrow \quad 3x^2 - 6x + 2 = 0$$

$$\Rightarrow \quad x = \frac{3 \pm \sqrt{3}}{3}$$

But $x = \frac{3-\sqrt{3}}{2} \notin [1, 2]$

Now, $f(1) = 1^3 - 3(1)^2 + 2(1) = 1 - 3 + 2 = 0$

$$f(2) = (2)^3 - 3(2)^2 + 2 \times 2$$

$$= 8 - 12 + 4 = 0$$

and $f\left(1+\frac{1}{\sqrt{3}}\right) = \left(1+\frac{1}{\sqrt{3}}\right)^3 - 3\left(1+\frac{1}{\sqrt{3}}\right)^2 + 2\left(1+\frac{1}{\sqrt{3}}\right)$

$$= 1^3 + \left(\frac{1}{\sqrt{3}}\right)^3 + 3 \times 1 \times \frac{1}{\sqrt{3}}\left(1+\frac{1}{\sqrt{3}}\right) - 3\left(1+\frac{1}{3}+\frac{2}{\sqrt{3}}\right) + 2 + \frac{2}{\sqrt{3}}$$

$$= 1 + \frac{1}{3\sqrt{3}} + \frac{3}{\sqrt{3}} + 1 - 3 - 1 - \frac{6}{\sqrt{3}} + 2 + \frac{2}{\sqrt{3}}$$

$$= -\frac{1}{\sqrt{3}} + \frac{1}{3\sqrt{3}} = -\frac{2}{3\sqrt{3}}$$

Hence, maximum value of $f(x)$ is 0.

2. Consider a 3×3 matrix with every element being equal to 1. Its only non-zero eigenvalue is _____.

Sol. (3) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

The characteristic equation is $|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 1 & 1 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \quad -\lambda^3 + 3\lambda^2 = 0 \Rightarrow \lambda = 3, 0, 0$$

The non-zero eigenvalue is 3.

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3. The Laplace transform of $f(t) = e^{2t} \sin(5t)\, u(t)$ is

(a) $\dfrac{5}{s^2 - 4s + 29}$ (b) $\dfrac{5}{s^2 + 5}$

(c) $\dfrac{s-2}{s^2 - 4s + 29}$ (d) $\dfrac{5}{s+5}$

Sol. (a) Given, $f(t) = e^{2t} \sin(5t)\, u(t)$

$$L[\sin u(t)(5t)] = \frac{5}{s^2 + 5^2} = \frac{5}{s^2 + 25}$$

$$\left[\therefore L[\sin at\, u(t)] = \frac{a}{s^2 + a^2}\right]$$

$$L[e^{2t} \sin(5t) u(t)] = \frac{5}{(s-2)^2 + 25} \qquad \left[\because L[e^{at} u(t)] = \frac{1}{s-a}\right]$$

$$L[e^{2t} \sin(5t) u(t)] = \frac{5}{s^2 - 4s + 4 + 25} = \frac{5}{s^2 - 4s + 29}$$

4. A function $y(t)$, such that $y(0) = 1$ and $y(1) = 3e^{-1}$, is a solution of the differential equation $\dfrac{d^2y}{dt^2} + 2\dfrac{dy}{dt} + y = 0$. Then, $y(2)$ is

(a) $5e^{-1}$ (b) $5e^{-2}$ (c) $7e^{-1}$ (d) $7e^{-2}$

Sol. (b) Given, $\dfrac{d^2y}{dt^2} + 2\dfrac{dy}{dt} + y = 0, y(0) = 1$ and $y(1) = 3e^{-1}$

$$(D^2 + 2D + 1)y = 0$$

Auxiliary equation is $m^2 + 2m + 1 = 0$

$\Rightarrow$ $(m+1)^2 = 0 \Rightarrow m = -1, -1$

Complementary function $= (C_1 + C_2 t)e^{mt}$

$= (C_1 + C_2 t) \cdot e^{-t}$

Solution of differential equation $=$ CF $= (C_1 + C_2 t)e^{-t}$

$\Rightarrow$ $y = (C_1 + C_2 t) \cdot e^{-t}$

To calculate the value of C_1 and C_2

At $t = 0, y = 1 \Rightarrow 1 = (C_1 + C_2 \times 0)e^0$

$1 = C_1$

At $t = 1, y = 3e^{-1} \Rightarrow 3e^{-1} = (1 + C_2 \cdot 1)e^{-1}$

$\Rightarrow$ $3 = 1 + C_2 \Rightarrow C_2 = 2$

At $t = 2, y = (1 + 2 \times 2)e^{-2}$

$y = 5e^{-2}$

5. The value of the integral

$$\oint_C \frac{2z+5}{\left(z - \frac{1}{2}\right)(z^2 - 4z + 5)}\, dz$$

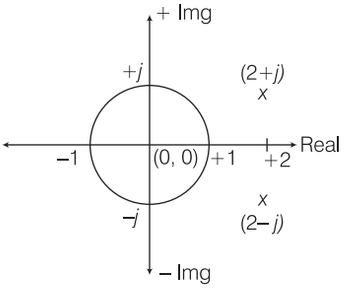
over the contour $|z| = 1$, taken in the anti-clockwise direction, would be

(a) $\dfrac{24\pi i}{13}$ (b) $\dfrac{48\pi i}{13}$

(c) $\dfrac{24}{13}$ (d) $\dfrac{12}{13}$

Sol. (b) To find $\oint_C \dfrac{2z+5}{\left(z - \frac{1}{2}\right)(z^2 - 4z + 5)} dz$ over the contour $|z| = 1$, in the anti-clockwise direction.

$|z| = 1$, is a contour with centre at (0, 0) and radius 1.



Poles of the given functions are

$$z = \frac{1}{2} \text{ and } z^2 - 4z + 5 = 0$$

$$z = \frac{1}{2} \text{ and } z = 2 \pm j$$

Now, according to Cauchy's residue theorem, if $f(z)$ is analytic at all points inside and on the boundary of a simple closed curve C except at finite number of poles lie inside C then, $\oint_C f(z)dz = 2\pi i$ [$\because$ Sum of residues of $f(z)$ at all its poles lies inside C].

So, $$f(z) = \lim_{z \to \frac{1}{2}} \left(z - \frac{1}{2}\right) \frac{(2z+5)}{\left(z - \frac{1}{2}\right)(z^2 - 4z + 5)}$$

$$= \lim_{z \to \frac{1}{2}} \frac{(2z+5)}{(z^2 - 4z + 5)}$$

$$= \frac{2 \times \frac{1}{2} + 5}{\left(\frac{1}{2}\right)^2 - 4 \times \frac{1}{2} + 5}$$

$$\text{Res. } f(z)_{z = \frac{1}{2}} = \frac{1+5}{\frac{1}{4} - 2 + 5} = \frac{6}{3 + \frac{1}{4}} = \frac{24}{13}$$

So, $$\oint_C \frac{2z+5}{\left(z - \frac{1}{2}\right)(z^2 - 4z + 5)} dz = 2\pi i \times \frac{24}{13} = \frac{48\pi i}{13}$$

6. The transfer function of a system is $\dfrac{Y(s)}{R(s)} = \dfrac{s}{s+2}$. The steady state output $y(t)$ is $A\cos(2t + \varphi)$ for the input $\cos(2t)$. The values of A and φ, respectively are

(a) $\dfrac{1}{\sqrt{2}}, -45°$ (b) $\dfrac{1}{\sqrt{2}}, +45°$

(c) $\sqrt{2}, -45°$ (d) $\sqrt{2}, +45°$

Sol. (b) Given, $\dfrac{Y(s)}{R(s)} = \dfrac{s}{s+2}$

$y(t) = A\cos(2t + \phi)$ and $r(t) = \cos 2t$

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$$H(s)=\frac{Y(s)}{R(s)}=\frac{s}{(s+2)}$$

$$H(j\omega)=\frac{j\omega}{j\omega+2} \Rightarrow |H(j\omega)|=\frac{\omega}{\sqrt{\omega^2+4}}$$

$$\angle H(j\omega)=90°-\tan^{-1}\left(\frac{\omega}{2}\right)$$

From $A\cos(2t+\phi)$, $\omega=2$

$$\therefore \quad |H(j\omega)|=\frac{2}{\sqrt{4+4}}=\frac{2}{2\sqrt{2}}=\frac{1}{\sqrt{2}}$$

$$|H(j\omega)|=90°-\tan^{-1}(1)=45°$$

$\because$ Hence, $A=1\times|H(j\omega)|_{\omega=2}=1\times\frac{1}{\sqrt{2}}=0.707$ and $\phi=+45°$

7. The phase cross-over frequency of the transfer function $G(s)=\frac{100}{(s+1)^3}$ in rad/s is

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) 3 (d) $3\sqrt{3}$

Sol. (a) Given, $G(s)=\frac{100}{(s+1)^3}$

$$G(j\omega)=\frac{100}{(1+j\omega)^3}=\frac{100}{1+(j\omega)^3+3(j\omega)^2+3j\omega}$$

$$=\frac{100}{(1-3\omega^2)+j(3\omega-\omega^3)}$$

$$=\frac{100[(1-3\omega^2)-j\omega(3-\omega^2)]}{[(1-3\omega)^2+\omega^2(3-\omega^2)^2]}$$

For phase crossover frequency ω_{ph},

$$\text{Img}[G(j\omega)]=0,$$

Hence, $\omega(3-\omega^2)=0$ or $\omega=0, \pm\sqrt{3}$

Therefore, $\omega_{ph}=\sqrt{3}$ rad/sec

8. Consider a continuous-time system with input $x(t)$ and output $y(t)$ given by

$$y(t)=x(t)\cos(t)$$

This system is

(a) linear and time-invariant

(b) non-linear and time-invariant

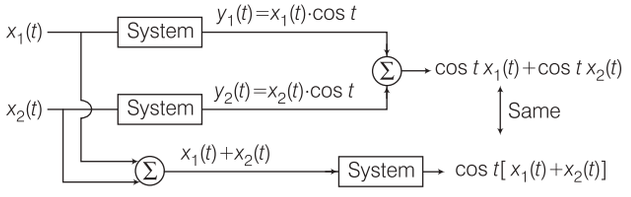
(c) linear and time-varying

(d) non-linear and time-varying

Sol. (c) Given, $y(t)=x(t)\cdot\cos(t)$ with input $=x(t)$

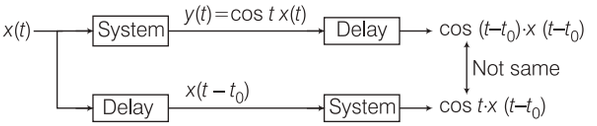
To check linearity

$\because$ Output $y(t)$ is zero for input $x(t)=0$



So, the above system is linear.

To check time variant/invariant



So, the above system is time variant.

So, system is linear and time variant.

9. The value of $\int_{-\infty}^{+\infty} e^{-t}\,\delta(2t-2)\,dt$, where $\delta(t)$ is the Dirac delta function, is

(a) $\frac{1}{2e}$ (b) $\frac{2}{e}$ (c) $\frac{1}{e^2}$ (d) $\frac{1}{2e^2}$

Sol. (a) Given, $\int_{-\infty}^{\infty} e^{-t}\delta(2t-2)\cdot dt$

$$\int_{-\infty}^{\infty} e^{-t}\delta(2t-2)\cdot dt=\frac{1}{2}\int_{-\infty}^{\infty} e^{-t}\delta(t-1)\cdot dt$$

$$\left[\because \text{From scaling property } i.e., \delta[a(t-t_1)]=\frac{1}{|a|}\delta(t-t_1)\right]$$

$$=\frac{1}{2}\left[e^{-1}\int_{-\infty}^{\infty}\delta(t-1)\cdot dt\right]$$

$$=\frac{1}{2}\times e^{-1}\times 1 \qquad \left[\because \frac{1}{2}\int_{-\infty}^{\infty}\delta(t-1)\cdot dt=1\right]$$

$$=\frac{1}{2e}$$

10. A temperature in the range of –40°C to 55°C is to be measured with a resolution of 0.1°C. The minimum number of ADC bits required to get a matching dynamic range of the temperature sensor is

(a) 8 (b) 10 (c) 12 (d) 14

Sol. (b) Resolution $=\frac{55-(-40)}{2^{n-1}}$

or $\quad 0.1=\frac{95}{2^{n-1}} \Rightarrow 2^{n-1}=950$

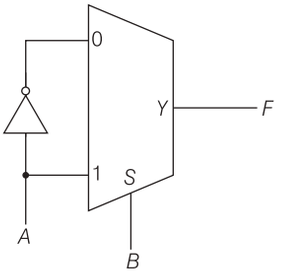
The minimum number of bits solved, n

$$2^{n-1}=2^9$$

$$n-1=9$$

$$n=10$$

11. Consider the following circuit which uses a 2-to-1 multiplexer as shown in the figure below. The Boolean expression for output F in terms of A and B is



(a) $A\oplus B$ (b) $\overline{A+B}$ (c) $A+B$ (d) $\overline{A\oplus B}$

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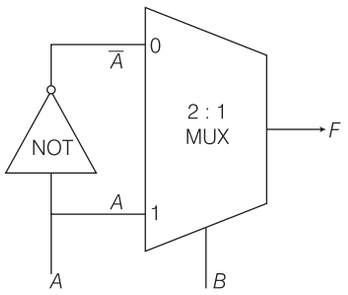
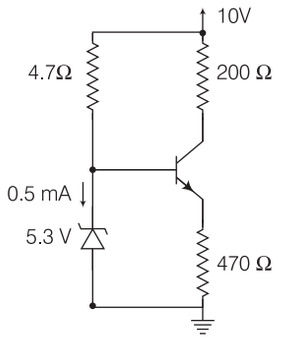
Sol. (d)



$$F = \overline{B} \cdot \overline{A} + B \cdot A$$
$$F = A \cdot B + \overline{A} \cdot \overline{B}$$
$$F = A \odot B$$
$$F = \overline{A \oplus B}$$

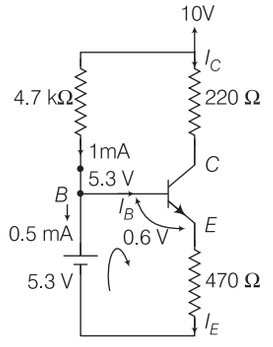
12. A transistor circuit is given below. The Zener diode breakdown voltage is 5.3 V as shown. Take base to emitter voltage drop to be 0.6 V. The value of the current gain β is _____.



Sol. (19) Given, in common-emitter transistor, $V_{BE} = 0.6\,V$

Apply KVL in the base-emitter loop

$$-5.3 + 0.6 + 470\, I_E = 0$$
$$\Rightarrow \quad 470 I_E = 4.7$$
$$\Rightarrow \quad I_E = 10 \text{ mA}$$



$$\text{Current in 4.7 k}\Omega \text{ resistor} = \frac{10 - 5.3}{4.7 \times 10^3} = 1 \text{ mA}$$

By KCL at node B,

$$\Rightarrow \quad I_B = 1 - 0.5 = 0.5 \text{ mA}$$

From $\quad I_E = I_B + I_C$

$$\Rightarrow \quad I_C = I_E - I_B = 10 - 0.5 = 9.5 \text{ mA}$$
$$\beta = \frac{I_C}{I_B} = \frac{9.5}{0.5} = 19$$

13. In cylindrical coordinate system, the potential produced by a uniform ring charge is given by $\varphi = f(r, z)$, where f is a continuous function of r and z. Let $\mathbf{E}$ be the resulting electric field. Then, the magnitude of $\nabla \times \mathbf{E}$

(a) increases with r (b) is 0

(c) is 3 (d) decreases with z

Sol. (b) A uniformly charged ring is specified. It can be considered as static. A static electric charge produces an electric field for which $\nabla \times \mathbf{E} = 0$.

14. A soft-iron toroid is concentric with a long straight conductor carrying a direct current I. If the relative permeability μ_r of soft-iron is 100, then the ratio of the magnetic flux densities at two adjacent points located just inside and just outside the toroid, is _____.

Sol. (100) Relative permeability of soft iron, $\mu_{r_i} = 100$

Let magnetic flux density inside the toroid = B_i

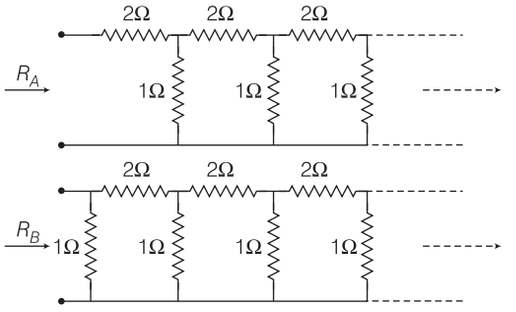
Magnetic flux density outside the toroid = B_o

So, $$\frac{B_i}{B_o} = \frac{\mu_i \cdot H}{\mu_o \cdot H}$$

[$\because$ Magnetic field intensity is same inside and outside]

$$\Rightarrow \quad \frac{B_i}{B_o} = \frac{\mu_o \mu_{r_i} \cdot H}{\mu_o \mu_{r_o} \cdot H} = \frac{\mu_{r_i}}{\mu_{r_o}}$$
$$= \frac{\text{Relative permeability of soft iron}}{\text{Relative permeability of soft air}}$$
$$\Rightarrow \quad \frac{B_i}{B_o} = \frac{100}{1} = 100$$

15. R_A and R_B are the input resistances of circuits as shown below. The circuits extend infinitely in the direction shown. Which one of the following statements is true?



(a) $R_A = R_B$ (b) $R_A = R_B = 0$

(c) $R_A < R_B$ (d) $R_B = R_A / (1 + R_A)$

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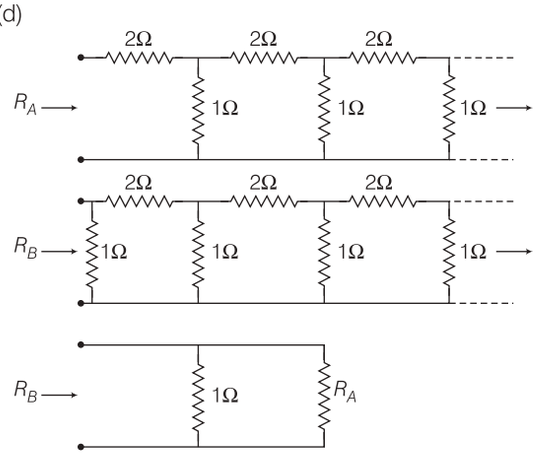
Sol. (d)



$$\Rightarrow \quad R_B = \frac{1 \times R_A}{1 + R_A}$$

$$\Rightarrow \quad R_B = \frac{R_A}{1 + R_A}$$

16. In a constant *V/f* induction motor drive, the slip at the maximum torque

(a) is directly proportional to the synchronous speed

(b) remains constant with respect to the synchronous speed

(c) has an inverse relation with the synchronous speed

(d) has no relation with the synchronous speed

Sol. (c) f_0 = nominal frequency

Synchronous speed, $\omega_s = \left(\frac{f}{f_0}\right)\omega_{s_0}$ ...(i)

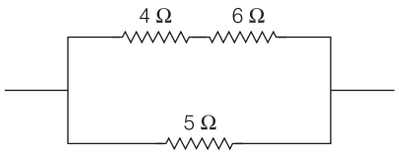
Slip at maximum torque, $S_{\max,T} = \left(\frac{f_0}{f}\right)\cdot\left(\frac{R_2}{X_{20}^1}\right)$ ...(ii)

From Eqs. (i) and (ii), we get

$$S_{\max,T} = \left(\frac{\omega_{s_0}}{\omega_s}\right)\cdot\left(\frac{R_2}{X_{20}^1}\right)$$

$$S_{\max,T} \propto \frac{1}{\omega_s}$$

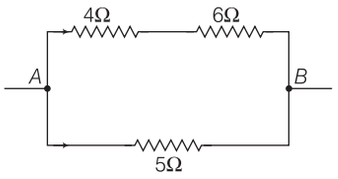
17. In the portion of a circuit shown, if the heat generated in 5 Ω resistance is 10 calories per second, then heat generated by the 4 Ω resistance, in calories per second, is _____.



Sol. (2) Given, heat generated in 5Ω resistance = 10 cal/s

$$\Rightarrow \quad \text{Power of } 5\,\Omega = 10 \text{ cal/s}$$

$$\text{Power} = I^2R \Rightarrow 10 = I^2 \times 5 \Rightarrow I^2 = 2 \Rightarrow I = \sqrt{2} \text{ A}$$



$$\Rightarrow \quad \text{Voltage, } V_{AB} = IR = \sqrt{2} \times 5 = 5\sqrt{2} \text{ V}$$

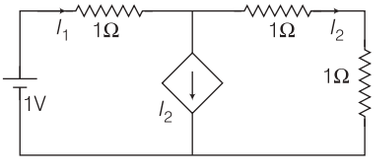
$$\text{Voltage across } 4\Omega = 5\sqrt{2} \times \frac{4}{4+6}$$

$$= 5\sqrt{2} \times \frac{4}{10} \quad \text{[voltage divider rule]}$$

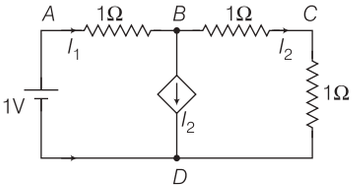
$$= 2\sqrt{2}$$

$$\text{Power of } 4\Omega = \frac{V^2}{R} = \frac{(2\sqrt{2})^2}{4} = \frac{8}{4} = 2 \text{ cal/s}$$

18. In the given circuit, the current supplied by the battery, in ampere, is _____.



Sol. (0.5)



Apply KCL at B, $I_1 = I_2 + I_2 = 2I_2$ ...(i)

or $$\frac{V_B - 1}{1} + \frac{V_B - 0}{1+1} + I_2 = 0$$ ...(ii)

or $$V_B - 1 + \frac{V_B}{2} + I_2 = 0$$

Also, $$V_B = 2I_2 = 2 \times \frac{I_1}{2} = I_1 \quad \left[\because I_2 = \frac{I_1}{2}, \text{ from Eq. (i)}\right]$$

On putting the values of V_B and I_2 in Eq. (ii), we get

$$I_1 - 1 + \frac{I_1}{2} + \frac{I_1}{2} = 0$$

or $$2I_1 = 1 \Rightarrow I_1 = \frac{1}{2} = 0.5 \text{ A}$$

19. In a 100 bus power system, there are 10 generators. In a particular iteration of Newton Raphson load flow technique (in polar coordinates), two of the *PV* buses are converted to *PQ* type. In this iteration,

(a) the number of unknown voltage angles increases by two and the number of unknown voltage magnitudes increases by two

(b) the number of unknown voltage angles remains unchanged and the number of unknown voltage magnitudes increases by two

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(c) the number of unknown voltage angles increases by two and the number of unknown voltage magnitudes decreases by two

(d) the number of unknown voltage angles remains unchanged and the number of unknown voltage magnitudes decreases by two

Sol. (b) Total number of buses = 100

Generator bus = 10 – 1 = 9

Load buses = 90

Slack bus = 1

If 2 buses are converted to *PQ* form *PV*, it will add 2 unknown voltages to iteration but unknown angles remains constant.

20. The magnitude of three-phase fault currents at buses *A* and *B* of a power system are 10 pu and 8 pu, respectively. Neglect all resistances in the system and consider the pre-fault system to be unloaded. The pre-fault voltage at all buses in the system is 1.0 pu. The voltage magnitude at bus *B* during a three-phase fault at bus *A* is 0.8 pu. The voltage magnitude at bus *A* during a three-phase fault at bus *B*, in pu, is _____.

Sol. (0.75) Given, prefault voltage, $V_{\text{Prefault}} = 1.0$ pu

Voltage at bus *B*, $V_B = 0.8$ pu

Voltage at bus *A*, $I_{fA} = 10$ pu

Fault current at bus *B*, $I_{fB} = 8$ pu

Use $V_B = V_{(\text{Prefault})} - Z_{12} I_{fB}$

$0.8 = 1.0 - Z_{12} \times 8$

$Z_{12} = 0.025$ pu

Similarly, $V_A = V_{(\text{Prefault})} - Z_{12} \times I_{fA}$

$= 1.0 - 0.025 \times 10$

$= 1.0 - 0.25$

Voltage at bus *A*, $V_A = 0.75$ pu

21. Consider a system consisting of a synchronous generator working at a lagging power factor, a synchronous motor working at an overexcited condition and a directly grid-connected induction generator. Consider capacitive VAr to be a source and inductive VAr to be a sink of reactive power. Which one of the following statements is true?

(a) Synchronous motor and synchronous generator are sources and induction generator is a sink of reactive power

(c) Synchronous motor and induction generator are sources and synchronous generator is a sink of reactive power

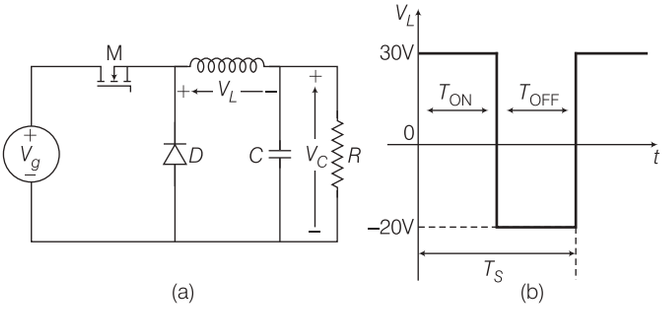
(c) Synchronous motor is a source and induction generator and synchronous generator are sinks of reactive power

(d) All are sources of reactive power

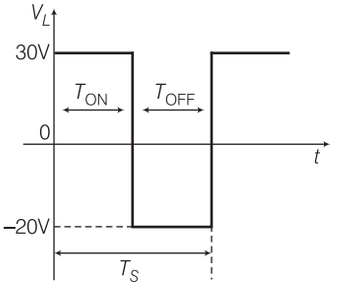
Sol. (a) Synchronous generator is source because it is working at a lagging power factor and supplying active power and lagging reactive power. Synchronous motor is working at an overexcited condition which means a leading PF operation so it will take active power and leading reactive power.

In other words, active power will be taken and lagging reactive power will be supplied. Although an induction generator generates active power but due to no any DC excitation in rotor, the lagging reactive power will be taken.

22. A buck converter, as shown in Figure (a) below, is working in steady state. The output voltage and the inductor current can be assumed to be ripple free. Figure (b) shows the inductor voltage V_L during a complete switching interval. Assuming all devices are ideal, the duty cycle of the buck converter is _____.



Sol. (0.4)



During $0 \le t \le T_{ON}$

$V_g - V_0 = V_L$

i.e., $V_g - V_0 = 30$ ...(i)

During $T_{ON} \le t \le T_S$

$-V_0 = V_L$

$\Rightarrow$ $V_0 = -V_L = -(-20)$

$V_0 = 20$ V

Duty cycle for buck converter,

$$\alpha = \frac{V_0}{V_g}$$

From Eq. (i) $V_g = 30 + V_0$

$= 30 + 20 = 50$ V

So, duty cycle, $\alpha = \frac{20}{50} = 0.4$

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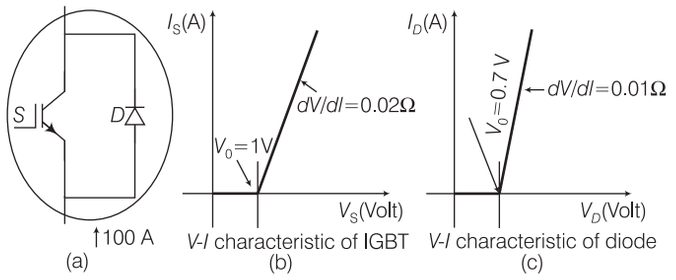
23. A steady DC current of 100 A is flowing through a power module (S, D) as shown in Figure (a). The V-I characteristics of the IGBT (S) and the diode (D) are shown in Figures (b) and (c), respectively. The conduction power loss in the power module (S, D), in watts, is _____.

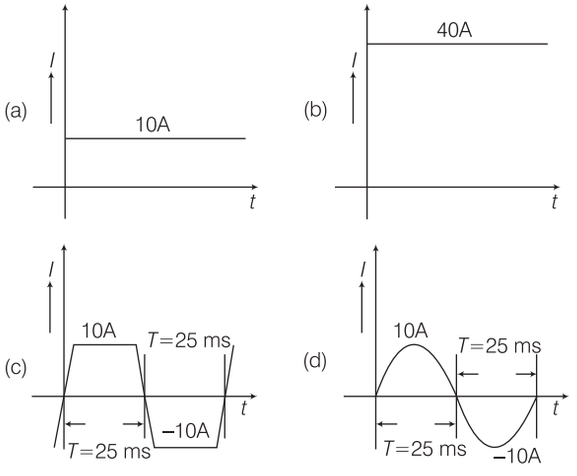


Sol. (170) No current flows through the IGBT. So, current flows only in diode.

Conduction power loss $= V_t I_{av} + I_{rms}^2 R_{on}$

$= 0.7 \times 100 + 100^2 \times 0.01 = 170$ W

24. A 4-pole, lap-connected, separately excited DC motor is drawing a steady current of 40 A while running at 600 rpm. A good approximation for the waveshape of the current in an armature conductor of the motor is given by



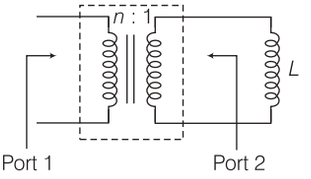
Sol. (c) With lap winding and 4 poles, number of parallel paths is equal to 4, with a total armature current of 40 A, current in each path and hence, current in each armature conductor is 10 A. It remains constant at 10 A as long as the conductor is in one path. When it goes into the next path (due to commutator action), the current in it reverses and becomes –10 A.

Assuming straight line commutation, the change from (+10 A) and (–10 A) is linear.

With 600 rpm, time for 1 revolution = 0.1 sec.

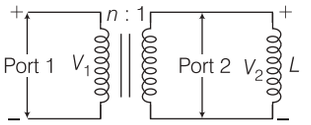
Time of a conductor to cover 1 pole-pitch = 0.1/ 4 = 25 ms. This is the width of one half cycle of conductor current.

25. If an ideal transformer has an inductive load element at port 2 as shown in figure below, the equivalent inductance at port 1 is



(a) nL (b) n^2L (c) $\frac{n}{L}$ (d) $\frac{n^2}{L}$

Sol. (b)



Given, $\frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{n}{1}$

Let the value of L at port 1 is L_1

So, $\frac{L}{L_1} = \left(\frac{V_2}{V_1}\right)^2 \Rightarrow L_1 = \left(\frac{V_1}{V_2}\right)^2 \times L = \left(\frac{n}{1}\right)^2 \times L$

$\Rightarrow$ $L_1 = n^2 L$

(2 Marks Questions)

26. Candidates were asked to come to an interview with 3 pens each. Black, blue, green and red were the permitted pen colours that the candidate could bring. The probability that a candidate comes with all 3 pens having the same colour is _____.

Sol. (0.0625) Probability that a candidate comes with all three pens having same color

$= P$(3 pens to be black) + P(3 pens to be blue) + P(3 pens to be green) + P(3 pens to be red)

$$= \frac{1}{4}\times\frac{1}{4}\times\frac{1}{4} + \frac{1}{4}\times\frac{1}{4}\times\frac{1}{4} + \frac{1}{4}\times\frac{1}{4}\times\frac{1}{4} + \frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}$$

$$= \frac{1}{64} + \frac{1}{64} + \frac{1}{64} + \frac{1}{64}$$

$$= \frac{1}{64}\times 4 = \frac{1}{16} = 0.0625$$

27. Let $S = \sum_{n=0}^{\infty} n\alpha^n$, where $|\alpha| < 1$. The value of α in the range $0 < \alpha < 1$, such that $S = 2\alpha$ is _____.

Sol. (0.30) $S = \sum_{n=0}^{\infty} n\alpha^n, |\alpha| < 1$

Given, α in $0 < \alpha < 1$ such that $S = 2\alpha$

$$2\alpha = \sum_{n=0}^{\infty} n\alpha^n = 0 + 1.\alpha^1 + 2.\alpha^2 + 3.\alpha^3 + 4.\alpha^4 + \ldots$$

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$\Rightarrow \quad 2\alpha = \alpha + 2\alpha^2 + 3\alpha^3 + 4\alpha^4$

[Neglect higher order terms since $|\alpha| < 1$]

$\Rightarrow \quad 4\alpha^3 + 3\alpha^2 + 2\alpha - 1 = 0$

On solving for α, we get

$\alpha = 0.30, -0.52 \pm 0.7373 = 0.30$

[$\because$ Only 0.30 lies between 0 and 1]

28. Let the eigenvalues of a 2×2 matrix A be 1, -2 with eigenvectors x_1 and x_2, respectively. Then, the eigenvalues and eigenvectors of the matrix $A^2 - 3A + 4I$ would respectively, be

(a) 2, 14; x_1, x_2

(b) 2, 14; $x_1 + x_2$, $x_1 - x_2$

(c) 2, 0; x_1, x_2

(d) 2, 0; $x_1 + x_2$, $x_1 - x_2$

Sol. (a) Eigenvalues of a 2×2 matrix A be 1, -2 with eigenvectors x_1 and x_2.

Eigenvalues of $A^2 \longrightarrow 1^2, (-2)^2 = 1, 4$

Eigenvalues of $3A \longrightarrow 3 \times 1, 3 \times (-2) = 3, -6$

Eigenvalues of $4I \longrightarrow 4$

Eigenvalues of

$$A^2 - 3A + 4I = [1 - 3 + 4], [(-2)^2 - 3(-2) + 4]$$

$$= 2, 14$$

$\because$ Eigenvectors remain same irrespective of the flotation applied on A, so eigenvectors of $A^2 - 3A + 4I$ are x_1 and x_2.

29. Let A be a 4×3 real matrix with rank 2. Which one of the following statement is true?

(a) Rank of $A^T A$ is less than 2

(b) Rank of $A^T A$ is equal to 2

(c) Rank of $A^T A$ is greater than 2

(d) Rank of $A^T A$ can be any number between 1 and 3

Sol. (a) Rank of a 4×3 matrix, $A = 2$

Since, rank of A^T = rank of A

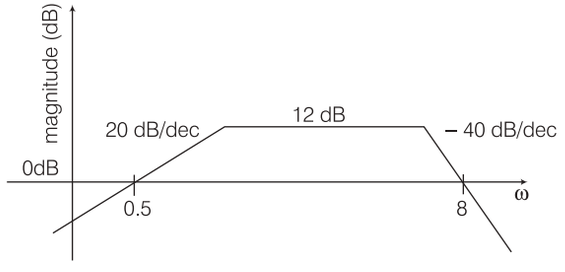
So, rank of $A^T = 2$

Now, Rank of $[A^T A] \le \min$ [Rank A^T, rank A]

$\Rightarrow \quad R[A^T A] \le \min [2, 2]$

$\Rightarrow$ Rank of $A^T A$ is less than 2.

30. Consider the following asymptotic Bode magnitude plot (ω is in rad/s).

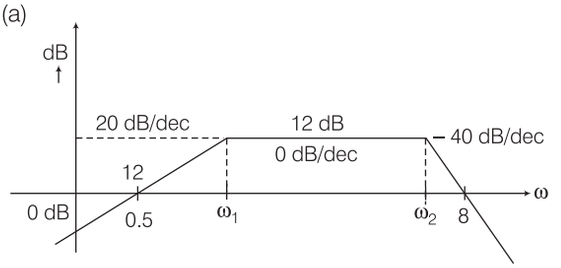


Which one of the following transfer functions is best represented by the given Bode magnitude plot?

(a) $\dfrac{2s}{(1 + 0.5s)(1 + 0.25s)^2}$ (b) $\dfrac{4(1 + 0.5s)}{s(1 + 0.25s)}$

(c) $\dfrac{2s}{(1 + 2s)(1 + 4s)}$ (d) $\dfrac{4s}{(1 + 2s)(1 + 4s)^2}$

Sol. (a)



Since, the starting slope is +20 dB/dec therefore, there is one zero at origin.

Since, there is a decrement of 20 dB/dec at ω_1 so, there is one pole at ω_1.

Since, there is a decrement of 40 dB/dec at ω_2 so, there is two pole at ω_2.

To find ω_1	**To find ω_2**
At $\omega = 0.5$,	At $\omega = 8$,
$y = mx + c$	$y = mx + c$
$\Rightarrow 0 = 20 \log 0.5 + c$	$\Rightarrow 0 = -40 \log 8 + c$
$\Rightarrow c = 6$	$\Rightarrow c = 36$
At $\omega = \omega_1$,	At $\omega = \omega_2$,
$y = mx + c$	$y = mx + c$
$\Rightarrow 12 = 20 \log \omega_1 + 6$	$\Rightarrow 12 = -40 \log \omega_2$
$\omega_1 = 2$ rad/s	$\Rightarrow \omega_2 = 4$ rad/s
To find K	
$20 \log K = 6 \Rightarrow K = 2$	

So, the transfer function is

$$G(s) = \frac{K \cdot s}{(1 + sT_1)(1 + sT_1)^2}$$

$\Rightarrow$

$$G(s) = \frac{2s}{\left(1 + s\frac{1}{2}\right)\left(1 + s\frac{1}{4}\right)^2}$$

$$= \frac{2s}{(1 + 0.5s)(1 + 0.25s)^2}$$

31. Consider the following state-space representation of a linear time-invariant system.

$$\dot{x}(t) = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} x(t), \; y(t) = c^T x(t), \; c = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

and $x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

The value of $y(t)$ for $t = \log_e 2$ is _____.

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Sol. (6) Given, $\dot{x}(t)=\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}x(t), y(t)=c^T x(t)$

$$c=\begin{bmatrix}1\\1\end{bmatrix} \text{ and } x(0)=\begin{bmatrix}1\\1\end{bmatrix}$$

$$A=\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}, x(t)=[L^{-1}(sI-A)^{-1}]x(0)$$

$$sI-A=\begin{bmatrix}s & 0\\ 0 & s\end{bmatrix}-\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}=\begin{bmatrix}s-1 & 0\\ 0 & s-2\end{bmatrix}$$

$$(sI-A)^{-1}=\frac{1}{(s-1)(s-2)}\begin{bmatrix}s-2 & 0\\ 0 & s-1\end{bmatrix}$$

$$(sI-A)^{-1}=\begin{bmatrix}1/(s-1) & 0\\ 0 & 1/(s-2)\end{bmatrix}$$

$$L^{-1}(sI-A)^{1}=L^{-1}\begin{bmatrix}1/(s-1) & 0\\ 0 & 1/(s-2)\end{bmatrix}=\begin{bmatrix}e^t & 0\\ 0 & e^{2t}\end{bmatrix}$$

$$x(t)\begin{bmatrix}e^t & 0\\ 0 & e^{2t}\end{bmatrix}x(0)=\begin{bmatrix}e^t & 0\\ 0 & e^{2t}\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}$$

$$\Rightarrow \quad x(t)=\begin{bmatrix}e^t\\ e^{2t}\end{bmatrix}$$

$$y(t)=[1 \quad 1]_{1\times 2}\begin{bmatrix}e^t\\ e^{2t}\end{bmatrix}$$

$$=1\times e^t+1\times e^{2t}=e^t+e^{2t}$$

$$\Rightarrow \quad y(t)|_{t=\log_e 2}=e^{\log_e 2}+e^{2\times\log_e 2}$$

$$=e^{\log_e 2}+e^{\log_e 4}$$

$$\Rightarrow \quad y(t)|_{t=\log_e 2}=2+4=6$$

32. Loop transfer function of a feedback system is $G(s)H(s)=\dfrac{s+3}{s^2(s-3)}$. Take the Nyquist contour in the clockwise direction. Then, the Nyquist plot of $G(s)H(s)$ encircles $-1+j0$

(a) once in clockwise direction
(b) twice in clockwise direction
(c) once in anti-clockwise direction
(d) twice in anti-clockwise direction

Sol. (c) Given, $G(s)H(s)=\dfrac{s+3}{s^2(s-3)}$

$$1+G(s)H(s)=1+\frac{(s+3)}{s^2(s-3)}$$

$$=\frac{s^2(s-3)+(s+3)}{s^2(s-3)}$$

$$\Rightarrow \quad 1+G(s)H(s)=\frac{s^3-3s^2+s+3\}}{s^2(s-3)\}} \begin{matrix}\rightarrow \text{Close loop poles}\\ \rightarrow \text{Open loop poles}\end{matrix}$$

Number of encirclements, $N=P-Z$

where, P = Number of open loops poles in RHS of s-plane.

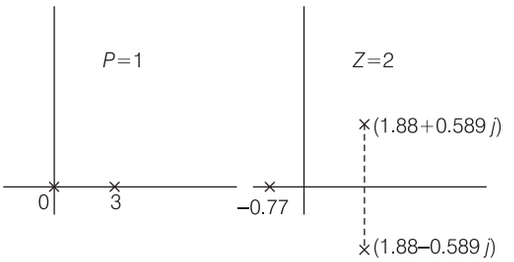
Z = Number of closed loops poles in RHS of s-plane.

For open loops poles P,

$$s^2(s-3)=0 \quad \Rightarrow \quad s=0,0,3 \qquad [\because P=3]$$

For closed loops poles Z,

$$s^3-3s^2+s+3=0 \quad \Rightarrow \quad s=-0.77, 1.88\pm 0.589j$$



So, $N=P-Z=1-2=-1$

Therefore, there will be once encirclement of Nyquist plot in anti-clockwise direction.

33. Given the following polynomial equation

$$s^3+5.5s^2+8.5s+3=0$$

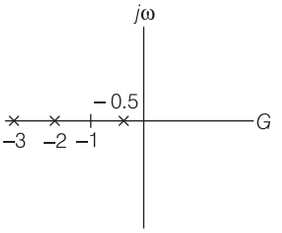
the number of roots of the polynomial, which have real parts strictly less than -1, is _____.

Sol. (2) Given, polynomial $s^3+5.5s^2+8.5s+3=0$

$$\Rightarrow \quad (s+2)(s^2+3.5s+1.5)=0$$

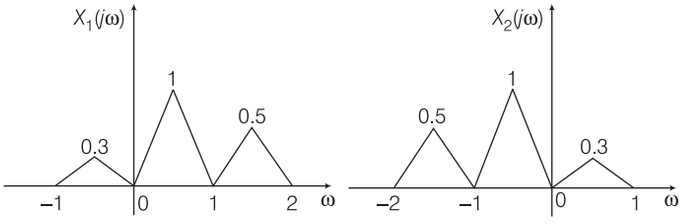
$$\Rightarrow \quad (s+2)(s+3)(s+0.5)=0$$

$$\Rightarrow \quad s=-2,-3,-0.5 \text{ are the roots.}$$



Clearly, number of roots having real part less than 1 are 2 *i.e.*, -3 and -2.

34. Suppose $x_1(t)$ and $x_2(t)$ have the Fourier transforms as shown below.



Which one of the following statements is true?

(a) $x_1(t)$ and $x_2(t)$ are complex and $x_1(t)\,x_2(t)$ is also complex with non-zero imaginary part
(b) $x_1(t)$ and $x_2(t)$ are real and $x_1(t)\,x_2(t)$ is also real
(c) $x_1(t)$ and $x_2(t)$ are complex but $x_1(t)\,x_2(t)$ is real
(d) $x_1(t)$ and $x_2(t)$ are imaginary but $x_1(t)\,x_2(t)$ is real

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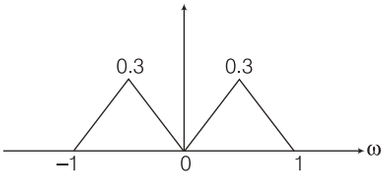
Sol. (c) Since, $X_1(j\omega)$ and $X_2(j\omega)$ are neither conjugate symmetric nor conjugate anti-symmetric, so $x_1(t)$ and $x_2(t)$ can not be purely real or imaginary so $x_1(t)$ and $x_2(t)$ are complex.



Now, $X_1(j\omega) \cdot X_2(j\omega) = X(j\omega)$

Clearly, $X_1(j\omega) \cdot X_2(j\omega)$ is conjugate symmetric as

$$X(j\omega) = X^*(-j\omega)$$

So, $x_1(t) \cdot x_2(t)$ is real.

35. The output of a continuous-time, linear time-invariant system is denoted by $T\{x(t)\}$, where $x(t)$ is the input signal. A signal $z(t)$ is called eigen-signal of the system T, when $T\{z(t)\} = \gamma z(t)$, where γ is a complex number, in general, and is called an eigenvalue of T. Suppose the impulse response of the system T is real and even. Which of the following statements is true?

(a) cos (t) is an eigen-signal but sin (t) is not
(b) cos (t) and sin (t) are both eigen-signals but with different eigenvalues
(c) sin (t) is an eigen-signal but cos (t) is not
(d) cos (t) and sin (t) are both eigen-signals with identical eigenvalues

Sol. (d) Given, that impulse response is real and even. Thus, $H(j\omega)$ will also be real and even.

$$e^{j\omega_0 t} \longrightarrow \boxed{\text{LTI System}} \longrightarrow H(j\omega_0)e^{j\omega_0 t}$$

$$e^{-j\omega_0 t} \longrightarrow \boxed{\text{LTI System}} \longrightarrow H(-j\omega_0)e^{-j\omega_0 t}$$

Since, $H(j\omega)$ is real and even thus,

$$H(j\omega_0) = H(-j\omega_0)$$

Now, cos(t) is input *i.e.*, $\dfrac{e^{jt} + e^{-jt}}{2}$ is input

Output will be

$$\frac{H(j1)e^{jt} + H(-j1)e^{-jt}}{2} = H(j1)\left[\frac{e^{jt} + e^{-jt}}{2}\right]$$
$$= H(j1)\cos(t)$$

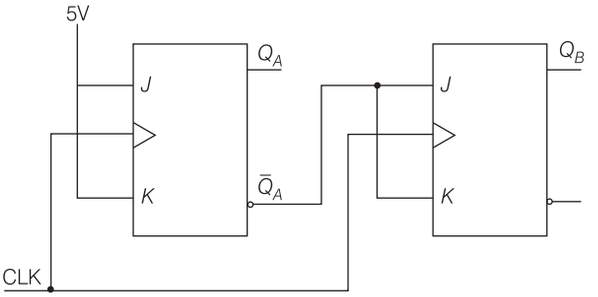
If sin(t) is input *i.e.*, $\dfrac{e^{jt} - e^{-jt}}{2}$ is input

Output will be

$$\frac{H(j1)e^{jt} - H(-j1)e^{-jt}}{2} = H(j1)\left[\frac{e^{jt} - e^{-jt}}{2}\right] = H(j1)\sin t$$

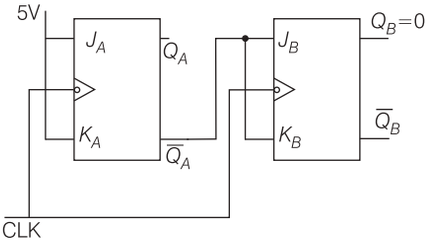
So, sin(t) and cos(t) are eigen-signals with same eigenvalues.

36. The current state $Q_A\, Q_B$ of a two JK flip-flop system is 00. Assume that the clock rise-time is much smaller than the delay of the JK flip-flop. The next state of the system is



(a) 00 (b) 01 (c) 11 (d) 10

Sol. (c)

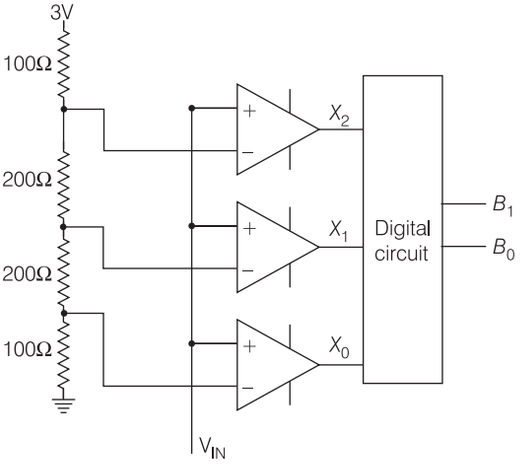


J	K	Q^+
0	0	No change
1	0	1 set
0	1	0 reset
1	1	$\bar{Q}$ Toggle

J_A	K_A	J_B	K_B	Q_A	Q_B
(Always High, 5V)		$(\bar{Q}_A)$	$(\bar{Q}_A)$	0	0
1	1	1	1	1	1
				Next state	

So, next state will be 11.

37. A 2-bit flash Analog to Digital Converter (ADC) is given below. The input is $0 \le V_{IN} \le 3$ V. The expression for the LSB of the output B_0 as a Boolean function of X_2, X_1 and X_0 is



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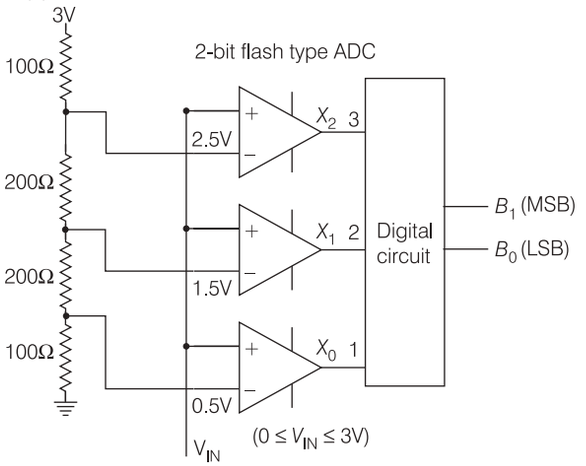
(a) $X_0\,[\overline{X_2 \oplus X_1}]$ (b) $\overline{X}_0\,[\overline{X_2 \oplus X_1}]$

(c) $X_0\,[X_2 \oplus X_1]$ (d) $\overline{X}_0\,[X_2 \oplus X_1]$

Sol. (a)



Truth Table of 2-bit Flash ADC

Analog input	Comparator outputs			Digital outputs	
V_{IN}	$X_2(3)$	$X_1(2)$	$X_0(1)$	B_1	B_0
0-1V	0	0	1	0	1
1-2V	0	1	1	1	0
2-3V	1	1	1	1	1

Operations

- With V_{IN} 0-1 V, output of X_0 will be high and that of X_2 and X_1 is low, so that digital output is 1 *i.e.,* 01.
- With V_{IN} 1-2 V, output of X_0 will be high and that of X_2 is low, so that digital output is 2 *i.e.,* 10.
- With V_{IN} 2-3 V, output of all X_0, X_1 and X_2 is high so that digital output is 3 *i.e.,* 11.

Expression for B_0 in SOP form is

$$= X_0\overline{X}_1\overline{X}_2 + X_0X_1X_2$$

$$= X_0(\overline{X}_1\overline{X}_2 + X_1X_2) = X_0(X_1 \odot X_2) = X_0(\overline{X_1 \oplus X_2})$$

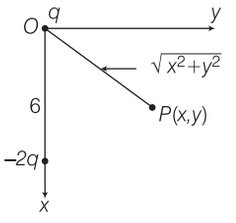
$$B_0 = X_0(\overline{X_1 \oplus X_2})$$

38. Two electric charges q and $-2q$ are placed at (0, 0) and (6, 0) on the x-y plane. The equation of the zero equipotential curve in the x-y plane is

(a) $x = -2$ (b) $y = 2$

(c) $x^2 + y^2 = 2$ (d) $(x + 2)^2 + y^2 = 16$

Sol. (d)



Potential at P due to q placed at origin

$$= \frac{q}{4\pi\varepsilon_0\sqrt{x^2 + y^2}}\ \text{V(ref: } \infty)$$

Potential at P due to $-2q$, placed at (6,0)

$$= \frac{-2q}{4\pi\varepsilon_0\sqrt{(x-6)^2 + y^2}}\ \text{V}$$

Net potential at

$$P = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{\sqrt{x^2 + y^2}} - \frac{2}{\sqrt{(x-6)^2 + y^2}}\right] = 0$$

$$\therefore \quad 4(x^2 + y^2) = (x-6)^2 + y^2 = x^2 + 36 - 12x + y^2$$

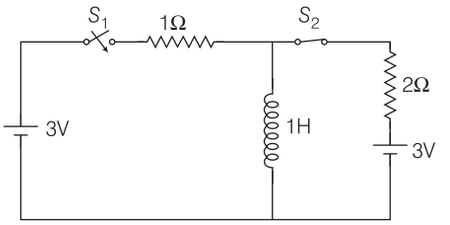
$$3x^2 + 3y^2 = 36 - 12x$$

$$x^2 + y^2 = 12 - 4x$$

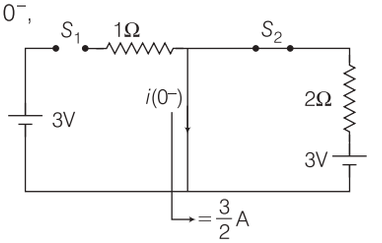
$$(x+2)^2 + y^2 = 16$$

(Equation of zero equipotential curve)

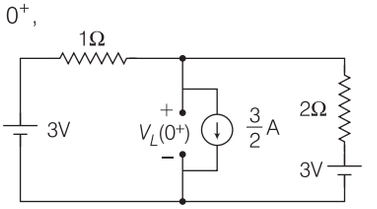
39. In the circuit shown, switch S_2 has been closed for a long time. At time $t = 0$, switch S_1 is closed. At $t = 0^+$, the rate of change of current through the inductor, in amperes per second, is _____.



Sol. (2) At $t = 0^-$,



At $t = 0^+$,



Nodal $\dfrac{(V_L(0^+)-3)}{1} + \dfrac{3}{2} + \dfrac{(V_L(0^+)-3)}{2} = 0$

$$\frac{2V_L(0^+) - 6 + 3 + V_L(0^+) - 3}{2} = 0$$

$$3V_L(0^+) = 6$$

$$V_L(0^+) = 2$$

So, $\dfrac{di(0^+)}{dt} = \dfrac{V_L(0^+)}{L} = \dfrac{2}{1} = 2$ A/s

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40. A three-phase cable is supplying 800 kW and 600 kVAr to an inductive load. It is intended to supply an additional resistive load of 100 kW through the same cable without increasing the heat dissipation in the cable, by providing a three-phase bank of capacitors connected in star across the load. Given the line voltage is 3.3 kV, 50 Hz, the capacitance per phase of the bank, expressed in microfarads, is _____.

Sol. (48) Initial load = $(800 + j600)$

Load after modification $= (900 + j600)$ to maintain same heat dissipation magnitude of power should be same.

Load with compensation

$$= (900 + j600 + \text{compensation}) = 900 + jx$$

Equating magnitude of power

$$800^2 + 600^2 = 900^2 + x^2$$

$$x = \sqrt{190000} = 435.889$$

Require reactive power = 435.889 kVAr

After compensation,

Reactive power to be compensated by capacitor to achieve this = 600 – 435.889 = 164.11 kVAr

$$\therefore \quad Q_{\text{bank}}/\text{ph} = \frac{164.11}{3} = 54.7 \text{ kVAr}$$

$$V/\text{ph} = \frac{3.3}{\sqrt{3}} = 1.9052 \text{ kV}$$

$$Q_c/\text{ph} = \frac{(V/\text{ph})^2}{X_c}$$

$$\Rightarrow \quad X_c = \frac{(V/\text{ph})^2}{(Q_c/\text{ph})}$$

$$X_c = \frac{(1.9052 \times 10^3)^2}{54.7 \times 10^3} = 66.36\,\Omega$$

$$\text{From } C = \frac{1}{2\pi f X_c}$$

$$= \frac{1}{2\pi \times 50 \times 66.36}$$

$$C = 47.96\ \mu\text{F}$$

41. A 30 MVA, 3-phase, 50 Hz, 13.8 kV, star-connected synchronous generator has positive, negative and zero sequence reactances, 15%, 15% and 5%, respectively. A reactance (X_n) is connected between the neutral of the generator and ground. A double line to ground fault takes place involving phases b and c, with a fault impedance of $j0.1$ pu The value of X_n (in pu) that will limit the positive sequence generator current to 4270 A is _____.

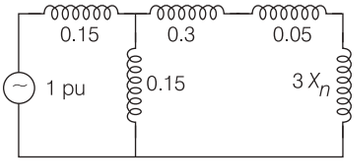
Sol. (1.1) Since, all the quantities are given in 'pu'.

$$\text{Positive sequence current in pu} = \frac{4270}{I_B}$$

$$\text{Base current, } I_B = \frac{30 \times 10^3}{\sqrt{3} \times 13.8} = 1255.109 \text{ A}$$

$$\text{Current in pu} = \frac{4270}{1255.109} = 3.4 \text{ pu}$$

The equivalent circuit can be drawn as



$$\text{Positive sequence current} = \frac{1}{X_{\text{eq}}} = 3.4$$

$$X_{\text{eq}} = \frac{1}{3.4} = 0.2941 \text{ pu}$$

$$0.15 + \frac{0.15(0.35 + 3X_n)}{3X_n + 0.5} = 0.2941$$

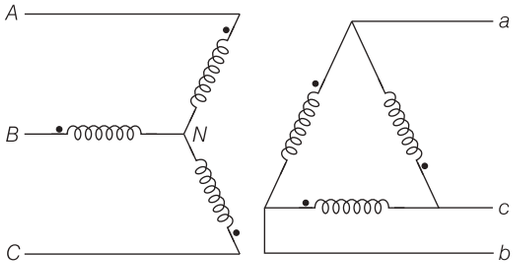
$$\frac{0.15(0.35 + 3X_n)}{3X_n + 0.5} = 0.1441$$

$$0.0525 + 0.45X_n = 0.4323X_n + 0.07205$$

$$0.0177X_n = 0.01955$$

$$X_n = 1.104 \text{ pu}$$

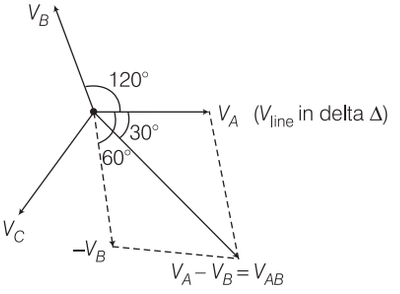
42. If the star side of the star-delta transformer shown in the figure is excited by a negative sequence voltage, then



(a) V_{AB} leads V_{ab} by 60° (b) V_{AB} lags V_{ab} by 60°

(c) V_{AB} leads V_{ab} by 30° (d) V_{AB} lags V_{ab} by 30°

Sol. (b) According to negative sequence phasors, the phasor diagram can be drawn as



V_{AB} lags V_{ab} by 30° (V_{line} in star Y)

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43. A single-phase thyristor-bridge rectifier is fed from a 230 V, 50 Hz, single-phase AC mains. If it is delivering a constant DC current of 10 A, at firing angle of 30°, then value of the power factor at AC mains is

(a) 0.87 (b) 0.9

(c) 0.78 (d) 0.45

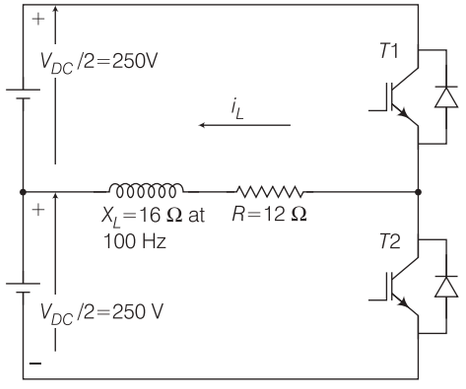
Sol. (c) Given, $1-\phi$ thyristor-bridge rectifier with, 230 V AC, 50 Hz, $I_0 = 10$ A (average output current) at firing angle $\alpha = 30°$.

Power factor on AC side = Distortion factor (g) × Fundamental Displacement Factor (FDF)

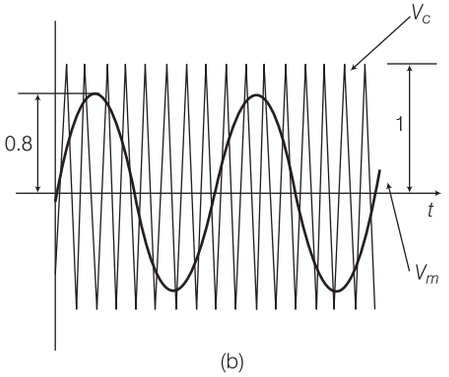
$$\Rightarrow \quad PF_{AC} = g \times FDF = \frac{2\sqrt{2}}{\pi} \times \cos\alpha$$

$$\Rightarrow \quad PF_{AC} = \frac{2\sqrt{2}}{\pi} \times \cos 30° = 0.779 \simeq 0.78$$

44. The switches $T1$ and $T2$ in Figure (a) are switched in a complementary fashion with sinusoidal pulse width modulation technique. The modulating voltage $V_m(t) = 0.8 \sin(200\pi t)$ V and the triangular carrier voltage (V_c) are as shown in Figure (b). The carrier frequency is 5 kHz. The peak value of the 100 Hz component of the load current (i_L), in ampere, is ____ .



(a)



(b)

Sol. (10) Modulation index, $m_a = \frac{\hat{V}_m}{\hat{V}_c} = \frac{0.8}{1} = 0.8$

Amplitude of the fundamental output voltage,

$$(\hat{V}_{AO})_1 = m_a \times \frac{V_{DC}}{2} = 0.8 \times 250 = 200 \text{ V}$$

From the given modulating voltage equation, it can be understood that $\omega_1 = 200\pi$ means, fundamental component frequency = 100 Hz

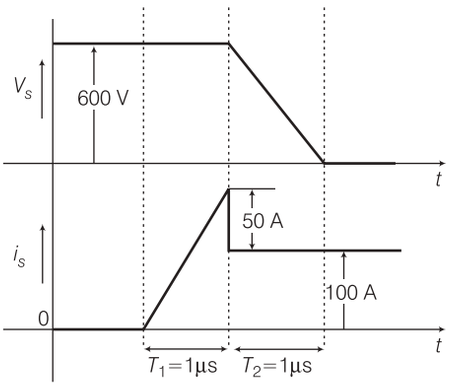
Load impedance at 100 Hz frequency,

$$Z_1 = \sqrt{R^2 + X^2}$$

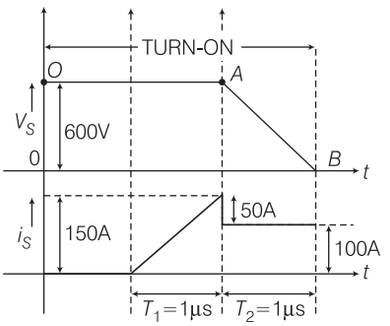
$$= \sqrt{12^2 + 16^2} = 20\,\Omega$$

$$\therefore \quad \hat{I}_{L1} = \frac{\hat{V}_{AO1}}{Z_1} = \frac{200}{20} = 10 \text{ A}$$

45. The voltage (V_s) across and the current (i_s) through a semiconductor switch during a turn-ON transition are shown in figure. The energy dissipated during the turn-ON transition, in mJ, is _____.



Sol. (75)



Energy = Power × Time

= Voltage × Current × Time

From point O to A,

$$\text{Energy dissipated} = 600 \times \left(\frac{1}{2} \times 150 \times 10^{-6}\right)$$

$$= 0.045 \text{ J} = 45 \text{ mJ}$$

Energy dissipated from A to B

$$= \left(\frac{1}{2} \times 600 \times 10^{-6}\right) \times 100$$

$$= 0.03 \text{ J} = 30 \text{ mJ}$$

Total energy dissipated = 45 + 30 = 75 mJ

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46. A single-phase 400 V, 50 Hz transformer has an iron loss of 5000 W at the rated condition. When operated at 200 V, 25 Hz, the iron loss is 2000 W. When operated at 416 V, 52 Hz, the value of the hysteresis loss divided by the eddy current loss is _____.

Sol. (1.44) Given 1– ϕ 400 V, 50 Hz transformer

Voltage (V)	Frequency (Hz)	Iron loss, P_i (W)
$400 = V_1$	$50 = f_1$	5000
$200 = V_2$	$25 = f_2$	2000
$416 = V_3$	$52 = f_3$	?

Iron loss = Eddy current loss + Hysteresis loss

$$\frac{V_1}{f_1} = \frac{400}{50} = 8,$$

$$\frac{V_2}{f_2} = \frac{200}{25} = 8,$$

$$\frac{V_3}{f_3} = \frac{416}{52} = 8$$

$\therefore$ In this operation, $\frac{V}{f}$ is constant

$\Rightarrow P_e$ (eddy current loss) $\propto f^2$ and P_h (Hysteresis loss) $\propto f$

At $V_1 = 400$ V and $f_1 = 50$ Hz

$P_{e1} = x f_1^2$ and $P_{h1} = yf$

$\Rightarrow P_{i1} = P_{e1} + P_{h1} \Rightarrow 5000 = xf_1^2 + yf$

$\Rightarrow 2500x + 50y = 5000$ ...(i)

At $V_2 = 200$ V and $f_2 = 25$ Hz

$P_{i2} = P_{e2} + P_{h2}$

$\Rightarrow 2000 = x \cdot (25)^2 + y \cdot (25) = 625x + 25y$

$\Rightarrow 625x + 25y = 2000$ ...(ii)

On solving the above Eqs. (i) and (ii) simultaneously, we get, $x = 0.8$ and $y = 60$

So, $P_{e1} = 2500 \times 0.8 = 2000$ W and $P_{h1} = 60 \times 50 = 3000$ W

Now, at $V_3 = 416$ V and $f_3 = 52$ Hz

$$\frac{P_{e3}}{P_{e1}} = \left(\frac{f_3}{f_1}\right) \Rightarrow \frac{P_{e3}}{2000} = \left(\frac{52}{50}\right) \Rightarrow P_{e3} = 2163.2 \text{ W}$$

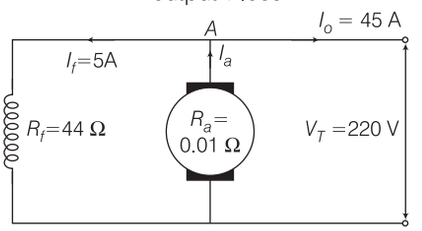
and $$\frac{P_{h3}}{P_{h1}} = \left(\frac{f_3}{f_1}\right) \Rightarrow \frac{P_{h3}}{3000} = \left(\frac{52}{50}\right) \Rightarrow P_{h3} = 3120 \text{ W}$$

Hence, the ratio of hysteresis loss and eddy current loss at 416 V, 52 Hz *i.e.*,

$$\frac{P_{h3}}{P_{e3}} = \frac{3120}{2163.2} = 1.44$$

47. A DC shunt generator delivers 45 A at a terminal voltage of 220 V. The armature and the shunt field resistances are 0.01 Ω and 44 Ω, respectively. The stray losses are 375 W. The percentage efficiency of the DC generator is _____.

Sol. (86.84) Efficiency, $\eta = \frac{\text{output}}{\text{output} + \text{loss}}$



$$I_f = \frac{220}{44} = 5 \text{ A and } I_a = I_f + I_o = 5 + 45 = 50 \text{ A}$$

Field loss = $5^2 \times 44 = 1100$ W

Armature loss = $50^2 \times 0.01 = 25$ W

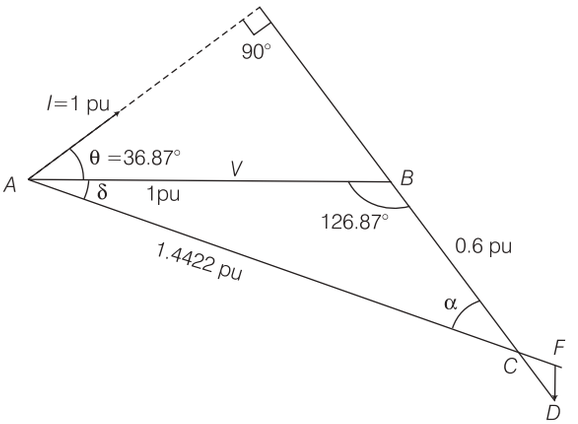
Total loss = $1100 + 25 + 375 = 1500$ W

Output = $45 \times 220 = 9900$ W

$$\eta = \frac{9900}{9900 + 1500} \times 100 = 86.84\%$$

48. A three-phase, 50 Hz salient-pole synchronous motor has a per-phase direct-axis reactance (X_d) of 0.8 pu and a per-phase quadrature-axis reactance (X_q) of 0.6 pu. Resistance of the machine is negligible. It is drawing full-load current at 0.8 pf (leading). When the terminal voltage is 1 pu, per-phase induced voltage, in pu, is _____.

Sol. (1.6078) Phasor diagram for **E** is constructed as below.



$$AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos 126.87°$$

$$= 1 + 0.36 + 1.2(0.6) = 2.08$$

$$AC = \sqrt{2.08} = 1.4422 \text{ pu}$$

$$\frac{1.4422}{0.8} = \frac{1}{\sin\alpha}$$

$\alpha = 33.7°$, so $\cos\alpha = 0.832$

$E = V\cos\delta + I_d X_d$

$\delta = 180° - 36.87° - (90° + \alpha)$

$\delta = 19.43°$ and $I_d = I_a \sin\theta = 0.831$

$E = 1 \times \cos 19.43° + 0.831 \times 0.8 = 1.6078$

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49. A single-phase, 22 kVA, 2200 V/220 V, 50 Hz, distribution transformer is to be connected as an auto transformer to get an output voltage of 2420 V. Its maximum kVA rating as an auto transformer is

(a) 22 (b) 24.2 (c) 242 (d) 2420

Sol. (c) Given, 1-ϕ, 22 kVA, 2200 V/220 V, 50 Hz distribution transformer.

Voltage rating = 2200 + 220 = 2420 V

The maximum (kVA) rating can be given as

$$(\text{kVA})_{\text{auto}} = (a_{2\,\text{winding}} + 1) \times \text{kVA}_{2\,\text{winding}}$$

where, $a_{2\,\text{winding}} = \dfrac{2200}{220} = 10$

so $(\text{kVA})_{\text{auto}} = (10 + 1) \times 22 = 242$

50. A single-phase full-bridge Voltage Source Inverter (VSI) is fed from a 300 V battery. A pulse of 120° duration is used to trigger the appropriate devices in each half-cycle. The rms value of the fundamental component of the output voltage, in volts, is

(a) 234 (b) 245 (c) 300 (d) 331

Sol. (a) Given, 1–ϕ , full bridge VSI

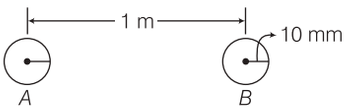
$$V_S = 300\,\text{V} \text{ and } 2d = 120° \Rightarrow d = 60° \text{ and } n = 1$$

$$V_{O1} = \frac{2\sqrt{2}}{\pi} . V_s \sin d = \frac{2\sqrt{2}}{\pi} \times 300 \times \sin 60°$$

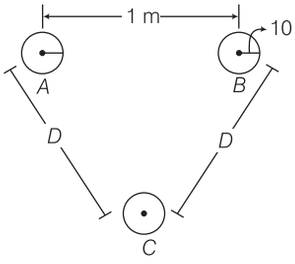
$$\Rightarrow \quad V_{O1} = 233.9 \approx 234\,\text{V}$$

51. A single-phase transmission line has two conductors each of 10 mm radius. These are fixed at a center-to-center distance of 1 m in a horizontal plane. This is now converted to a three-phase transmission line by introducing a third conductor of the same radius. This conductor is fixed at an equal distance D from the two single-phase conductors. The three-phase line is fully transposed. The positive sequence inductance per phase of the three-phase system is to be 5% more than that of the inductance per conductor of the single-phase system. The distance D, in metres, is _____.

Sol. (1.427) In first case, when two conductors are at distance 1 m apart.



In second case, when third conductors is introduced at distance D from the two 1-ϕ conductors.



$$L_1 = 2 \times 10^{-7} \ln \frac{D}{r}$$

$$= 2 \times 10^{-7} \ln\left(\frac{100}{0.7788}\right) = 0.97\,\mu\text{H/m}$$

$$L_2 = 1.05 \times 0.97 = 1.0185\ \mu\text{H/m}$$

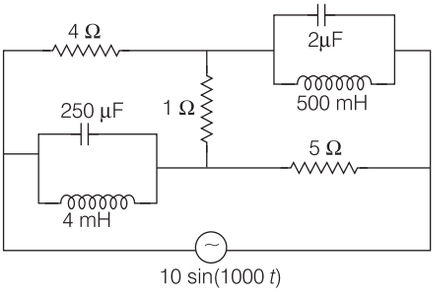
$$L_2 = 2 \times 10^{-7} \ln\left(\frac{\sqrt[3]{D^2 \times 100}}{0.7788}\right)$$

$$\ln\left(\frac{\sqrt[3]{D^2 \times 100}}{0.7788}\right) = \frac{1.0185 \times 10^{-6}}{2 \times 10^{-7}} = 5.0925$$

$$e^{5.0925} = \frac{\sqrt[3]{100D^2}}{0.7788}$$

$$D = 142.7\ \text{cm} = 1.427\ \text{m}$$

52. In the circuit shown below, the supply voltage is $10 \sin (1000t)$ V. The peak value of the steady state current through the 1 Ω resistor, in amperes, is _____.



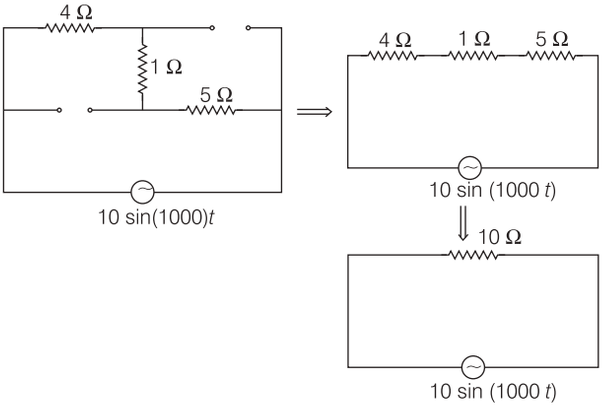
Sol. (1) $X_C = \dfrac{1}{j\omega C} = \dfrac{1}{j \times 1000 \times 250 \times 10^{-6}} = \dfrac{4}{j} = -4j$

$$X_L = j\omega L = j \times 1000 \times 4 \times 10^{-3} = 4j$$

$$Z = \frac{(-4j)(4j)}{-4j + 4j} = \infty \ \textit{i.e.,}\ \text{open circuit}$$

Similarly for the combination of 2 μF and 500 mH, $Z = \infty$.

Circuit reduces to



So, peak value of I through $1\,\Omega = \dfrac{10}{10} = 1\,\text{A}$

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53. A DC voltage with ripple is given by

$V(t) = [100 + 10 \sin(\omega t) - 5 \sin(3\omega t)]$ V.

Measurements of this voltage $V(t)$, made by moving-coil and moving-iron voltmeters, show readings of V_1 and V_2, respectively. The value of $V_2 - V_1$, in volts, is _____.

Sol. (0.312) DC voltage, $V(t) = [100 + 10\sin\omega t - 5\sin 3\omega t]$

Measurement by moving-coil V_1

V_1 = Average value of $V(t) = 100$ V

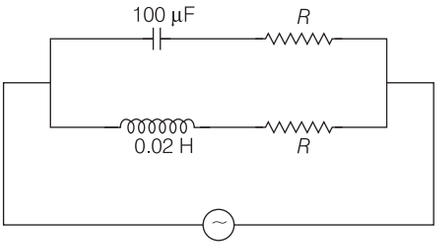
Measurement by moving-iron V_2

$$V_2 = \text{RMS value of } V(t) = \sqrt{100^2 + \left(\frac{10}{\sqrt{2}}\right)^2 + \left(\frac{-5}{\sqrt{2}}\right)^2}$$

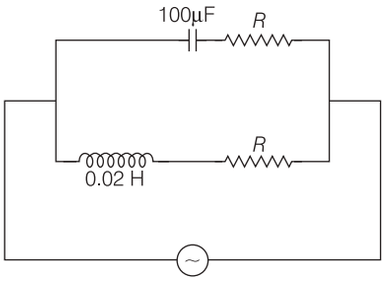
$\Rightarrow \quad V_2 = 100.312$ V

$V_2 - V_1 = 100.312 - 100 = 0.312$ V

54. The circuit below is excited by a sinusoidal source. The value of R, in Ω for which the admittance of the circuit becomes a pure conductance at all frequencies is _____.



Sol. (14.14)



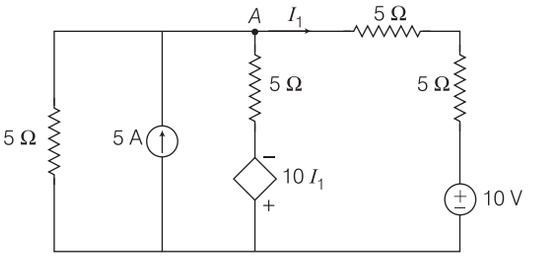
$$Y = (R_e) + j(I_m)$$

$\because \quad j(I_m) = 0$

$$Y = R = \sqrt{\frac{L}{C}} = \sqrt{\frac{0.02}{100 \times 10^{-6}}} = 14.14$$

If we choose $R = \sqrt{\frac{L}{C}}$, then irrespective of frequency, the network is purely resistive (admittance becomes conductance), *i.e.*, voltage and current are always in phase and pf is always unity.

55. In the circuit shown below, the node voltage V_A is _____ V.



Sol. (11.42) Apply KCL at A, we get

$$\frac{V_A - 0}{5} + \frac{V_A + (10I_1)}{5} + \frac{V_A - 10}{5+5} = 5$$

$$\Rightarrow \quad \frac{V_A}{5} + \frac{V_A + (10I_1)}{5} + \frac{V_A - 10}{10} = 5$$

$$\Rightarrow \quad 2V_A + 2(V_A + 10I_1) + (V_A - 10) = 50$$

Also, $I_1 = \frac{V_A - 10}{10}$

$$\Rightarrow \quad 2V_A + 2\left[V_A + 10 \times \frac{(V_A - 10)}{10}\right] + (V_A - 10) = 50$$

$$2V_A + 2(V_A + V_A - 10) + V_A - 10 = 50$$

$$\Rightarrow \quad 6V_A - 20 + V_A - 10 = 50$$

$$\Rightarrow \quad 7V_A = 80$$

$$\Rightarrow \quad V_A = \frac{80}{7} = 11.42 \text{ V}$$

General Aptitude (GA) Questions

(1 *Mark Questions*)

56. The man who is now Municipal Commissioner worked as _____.

(a) the security guard at a university
(b) a security guard at the university
(c) a security guard at university
(d) the security guard at the university

Sol. (b) Options (a) and (d) are incorrect due to word 'the security'.

57. Nobody knows how the Indian cricket team is going to cope with the difficult and seamer-friendly wickets in Australia.

Choose the option which is closest in meaning to the underlined phrase in the above sentences.

(a) put up with
(b) put in with
(c) put down to
(d) put up against

Sol. (a)

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58. Find the odd one in the following group of words.

mock, deride, praise, jeer

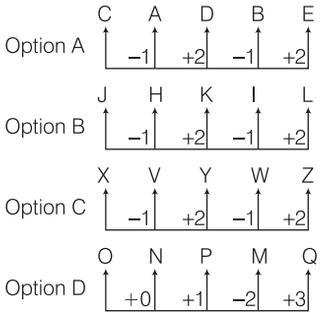
(a) mock (b) deride
(c) praise (d) jeer

Sol. (c) 'mock', 'deride' and jar are synonyms of mockery. Therefore, jeer is the odd one.

59. Pick the odd one from the following options.

(a) CADBE (b) JHKIL
(c) XVYWZ (d) ONPMQ

Sol. (d)



60. In a quadratic function, the value of the product of the roots (α, β) is 4. Find the value of

$$\frac{\alpha^n + \beta^n}{\alpha^{-n} + \beta^{-n}}$$

(a) n^4 (b) 4^n
(c) 2^{2n-1} (d) 4^{n-1}

Sol. (b) Given, product of roots (α, β) is 4,

$$i.e., \ \alpha \cdot \beta = \text{Value of } \frac{\alpha^n + \beta^n}{\alpha^{-n} + \beta^{-n}} = \frac{\alpha^n + \beta^n}{\frac{1}{\alpha^n} + \frac{1}{\beta^n}}$$

$$\Rightarrow \quad \frac{\alpha^n + \beta^n}{\frac{\beta^n + \alpha^n}{\alpha^n \cdot \beta^n}} = \frac{\alpha^n + \beta^n}{\alpha^n + \beta^n} \times \alpha^n \cdot \beta^n$$

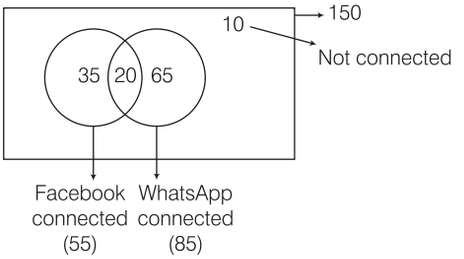
$$\Rightarrow \quad \alpha^n \cdot \beta^n = (\alpha \cdot \beta)^n = 4^n$$

(2 Marks Questions)

61. Among 150 faculty members in an institute, 55 are connected with each other through Facebook and 85 are connected through WhatsApp. 30 faculty members do not have Facebook or WhatsApp accounts. The number of faculty members connected only through Facebook accounts is _____.

(a) 35 (b) 45
(c) 65 (d) 90

Sol. (a) **Case 1** Let 30 faculty members do not have facebook account.



Number of faculty members having Facebook account

$= 150 - 30 = 120$

Number of faculty members connected only Facebook

$= 120 - 85 = 35$

Case 2 Let 30 faculty members do not have WhatsApp account.

Number of faculty members having WhatsApp account

$= 150 - 30 = 120$

Number of faculty members connected only WhatsApp

$= 120 - 55 = 65$

So, number of faculty members connected by both Facebook and WhatsApp $= 85 - 65 = 20$

62. Computers were invented for performing only high-end useful computations. However, it is no understatement that they have taken over our world today. The internet, for example, is ubiquitous. Many believe that the internet itself is an unintended consequence of the original invention. With the advent of mobile computing on our phones, a whole new dimension is now enabled. One is left wondering, if all these developments are good or, more importantly, required.

Which of the statement(s) below is/are logically valid and can be inferred from the above paragraph?

I. The author believes that computers are not good for us.

II. Mobile computers and the internet are both intended inventions.

(a) Only I (b) Only I
(c) Both I and II (d) Neither I nor II

Sol. (d)

63. All hill-stations have a lake. Ooty has two lakes.

Which of the statement(s) below is/are logically valid and can be inferred from the above sentences?

I. Ooty is not a hill-station.

II. No hill-station can have more than one lake.

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

Sol. (d)

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64. In a 2×4 rectangle grid shown below, each cell is a rectangle. How many rectangles can be observed in the grid?

(a) 21 (b) 27 (c) 30 (d) 36

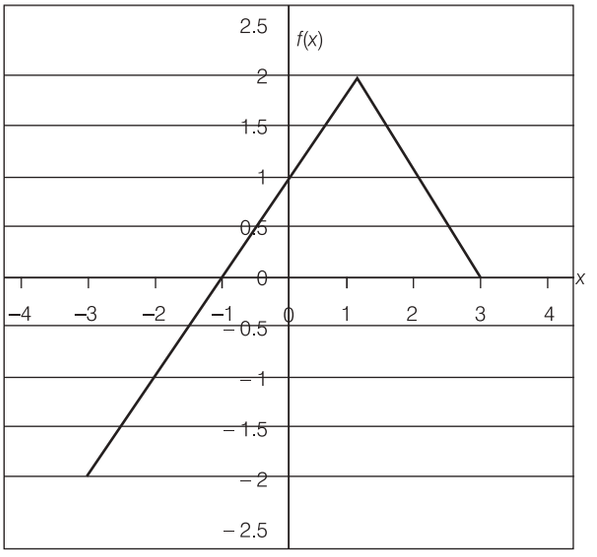
Sol. (c) As in the given figure, there are 3 horizontal lines and 5 vertical lines, the two horizontal and at least two vertical lines will be chosen to make a rectangle.

$\therefore$ Number of rectangles $= {}^5C_2 \times {}^3C_2$

$$= \frac{5!}{3! \times 2!} \times \frac{3!}{1! \times 2!} = \frac{5 \times 4 \times 3!}{3! \times 2} \times \frac{3 \times 2!}{2!}$$

$$= 10 \times 3 = 30$$

65. Choose the correct expression for $f(x)$ given in the graph.



(a) $f(x) = 1 - |x - 1|$ (b) $f(x) = 1 + |x - 1|$ (c) $f(x) = 2 - |x - 1|$ (d) $f(x) = 2 + |x - 1|$

Sol. (c) At $x = 3$, the graph $f(x)$ must be equal to zero.

So, from option (c), $f(x) = 2 - |x - 1|$

$$= 2 - |3 - 1|$$

$$= 2 - 2$$

$$f(x) = 0$$

Similarly at $x = 1$, $f(x)$ must be equal to 2.

$$f(x) = 2 - |1 - 1| = 2$$